Work – Energy Pack 1

When a force acts on an object, and that object moves, there is a concept involved called "Work". Work is vital to understanding Energy, our next topic. It turns out that Work is a means by which energy is transferred. But before we can discuss Energy and transfers, we have to understand the concept of Work.

In Physics, "Work" means something different than its everyday language usage. In the simplest terms, Work is a Force F times a Displacement d. Force and Displacement are both vectors, and, there are two different ways to multiply vectors in math. The technique we will use in the case of Work is called the "Dot Product", and it is written as $W = \vec{F} \bullet \vec{d}$. The result (Work) is a scalar, meaning no longer a vector.

Work done by a single force

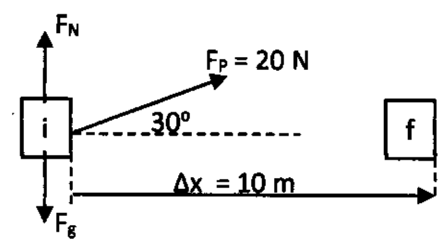
To multiply two vectors as a Dot Product, you place them tail to tail and multiply their magnitudes together. Remember, magnitudes are always positive. You then multiply that result by the cosine of the angle between the two vectors. Applied to Work, if you have a Force vector $\vec{F}$ and a Displacement vector $\vec{d}$, then Work is the Dot Product of $\vec{F}$ and $\vec{d}$, meaning is $W = \vec{F} \bullet \vec{d} = |F||d|\cos\beta$. Here's an example:

Example 1:

A force of 20 N, angled 30° above horizontal to the right, pulls an object horizontally over a level frictionless surface in a straight line for 10 m. How much work does the pulling force do on the object?

Here's a picture showing the applicable vectors of this scenario:

This drawing is a combined FBD and initial / final drawing. It is the drawing we will use for all this work – energy stuff. Since the forces are the same at both the initial and final positions, you only need to show the FBD at one position, usually the initial.



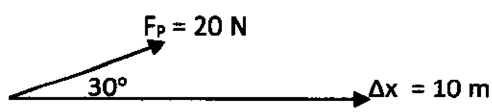
The two vectors of significance to finding Work done by F_P on the object are F_P and Δx.

Work is: $$W = \vec{F} \bullet \vec{d}$$

In this example, "F" is F_P and "d", the displacement vector, is Δx since it's moving horizontally.

So in this case: $$W = \vec{F}_P \bullet \overrightarrow{\Delta x} = |F_P||\Delta x|\cos\beta$$

To find θ, draw the F_P and Δx vectors tail to tail as follows



This shows the angle θ between the two vectors to be 30°, making Work:

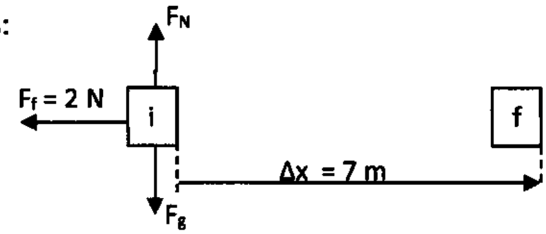
$$W = 20 \cdot 10 \cdot \cos(30^0) = 173.21 \text{ N} \cdot \text{m}$$

Note that you can write this as W = (F_P)cos(β)Δx, which is the same thing as $F_{Px}\Delta x$. So another way to think of the Work Dot Product is "the component of force parallel to displacement times that displacement."

Example 2:

A block slides to a stop due to friction along a level surface. The force of friction acting on the block is 2 N and it takes 7 m for the block to stop. How much work does the force of friction do on the block?

Here's a drawing of the applicable force and displacement vectors:



Work is found by:

$$W = \vec{F} \bullet \vec{d}$$
$$W = \vec{F}_f \bullet \overrightarrow{\Delta x} = |F_f||\Delta x|\cos\beta$$

Draw the F_P and Δx vectors tail to tail yields:

which makes Work:

$$W = |F_f||\Delta x|\cos\beta$$
$$W = (2)(7)\cos(180^\circ)$$
$$W = -14\ \text{N}\cdot\text{m}$$

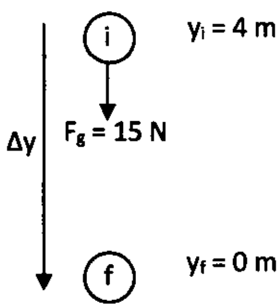
Negative Work results when the force in question (or a component of it) points opposite to the direction of displacement.

The notion of Work as "the component of force parallel to displacement times that displacement" is still valid; however, you must then make Work negative when the force is in the opposite direction as displacement.

You can also correctly keep track of the sign of Work in this approach by saying F_f points left, so it must be negative and Δx points right so it must be positive. And positive times negative yields a negative.

Example 3:

A 1.5 kg rock is dropped from of height of 4 m. How much work does gravity do on the ball as it falls to the ground?

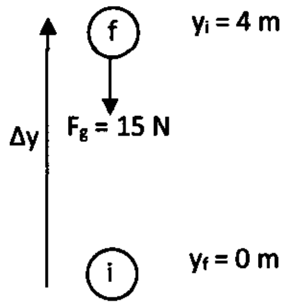


$$W = \vec{F} \bullet \vec{d}$$
$$W = \vec{F}_g \bullet \overrightarrow{\Delta y} = |F_g||\Delta y|\cos\beta$$
$$W = |F_g||\Delta y|\cos\beta$$
$$W = (15)(4)\cos(0^\circ)$$
$$W = 60J$$

Since F_g and Δy point in the same direction, β = 0°.

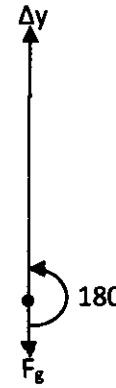
Example 4:

A 1.5 kg rock is thrown straight up from the ground to a height of 4 m. How much work does gravity do on the ball as it rises to 4 m?



$$W = \vec{F} \bullet \vec{d}$$
$$W = \vec{F}_g \bullet \overrightarrow{\Delta y} = |F_g||\Delta y|\cos\beta$$
$$W = |F_g||\Delta y|\cos\beta$$
$$W = (15)(4)\cos(180^\circ)$$
$$W = -60 N \cdot m$$

Since F_g and Δy point in opposite directions, β = 180°.



Work done by a total force

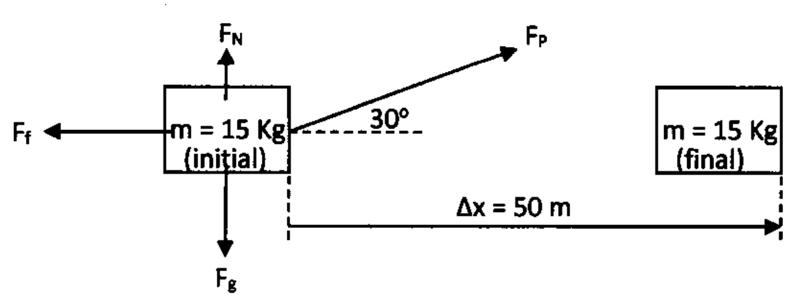
That's how to calculate work done by one particular force. However, we frequently want to find the total Work (W_T) done on an object(s) by all the forces acting on that object(s). To do this, you first identify the direction of displacement, frequently either Δx or Δy. Then you find the total force parallel to that direction, frequently F_{TX} or F_{TY}. All the techniques you have learned to determine F_{TX} or F_{TY} apply here as well. You calculate W_T by multiplying the F_T parallel to the displacement (ie F_{TX} orF_{TY}) by the displacement (ie Δx or Δy), making W_T positive if F_T points in the same direction as Displacement and negative if F_T points in the opposite direction.

Here's an example, Example 6:

A girl pulls a 15 Kg sled across the level snow with a rope which is angled 30° above horizontal. The girl pulls on the rope with a force of 40 N. The coefficient of friction between the sled and the snow is 0.11. What is the total work done on the sled as the girl pulls the sled for 50 m?

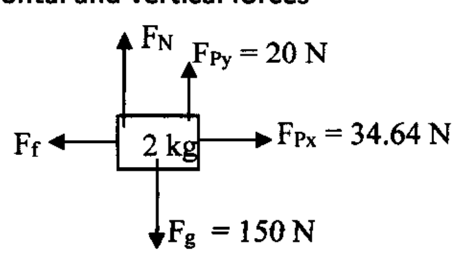
Solution:

Step 1: Make a FBD and initial/final drawing. The problem doesn't specify which way the sled is being pulled, so pick one. Also, include the displacement, here Δx.



Step 2: Calculate F_g, do trig on the angled pull, redraw FBD with only horizontal and vertical forces

$F_g = ma_g$ | $F_{Px} = (F_P)\cos(\theta)$ | $F_{Py} = (F_P)\sin(\theta)$
$F_g = (15\text{ kg})(-10\text{ m/s}^2)$ | $F_{Px} = (40\text{ N})\cos(30^\circ)$ | $F_{Py} = (40\text{ N})\sin(30^\circ)$
$F_g = -150$ N | $F_{Px} = 34.64$ N | $F_{Py} = 20$ N



Step 3: Determine the total force parallel to the direction of displacement.

Since the motion is horizontal, the displacement is Δx, making F_{TX} the total force parallel to the direction of displacement.

$F_{TX} = F_{Px} + F_f$ (from FBD)
$F_{TX} = 34.64 + -14.3$
$F_{TX} = 20.34$ N

$F_f = \mu F_N$
$F_f = 0.11(130)$
$F_f = 14.3$ N

$F_{TY} = 0$ (from 1st Law)
$F_{TY} = F_{Py} + F_N + F_g$ (from FBD)
$0 = 20 + F_N - 150$
$F_N = 130$ N

Step 4: Solve for total work.

Since $F_{TX} = 20.34$ N, it's a positive number, meaning 20.34 N to the right. Displacement (Δx) is also to the right, thus also positive: $\Delta x = 50$ m. This yields:

$$W_T = F_{TX} \cdot \Delta x$$
$$W_T = 20.34\text{ N})(50\text{ m})$$
$$W_T = 1017\text{ N}\cdot\text{m}$$

Another way to arrive at the fact that W_T must be positive is that the total force is pointing in the same direction as the displacement.

As before, you can analyze the work done by any of the individual forces. Here, I show this using Dot Products

$$W_{F_N} = |F_N||\Delta x|\cos\theta = (130)(50)\cos(90^\circ) = 0\text{ N}\cdot\text{m}$$
$$W_{F_g} = |F_g||\Delta x|\cos\theta = (150)(50)\cos(90^\circ) = 0\text{ N}\cdot\text{m}$$
$$W_{F_P} = |F_P||\Delta x|\cos\theta = (40)(50)\cos(30^\circ) \cong 1732\text{ N}\cdot\text{m}$$
$$W_{F_f} = |F_f||\Delta x|\cos\theta = (14.3)(50)\cos(180^\circ) = -715\text{ N}\cdot\text{m}$$

So the total work is the sum of the work done by each individual force = 0 + 0 + 1732 – 715 = 1017 J

Work Concept Summary:

To get work, you must have two things:

1) A force or a component of a force acting parallel to the direction of displacement. The force (or component) can be acting in the same direction, or the exact opposite direction of displacement.
2) Displacement – meaning the object must actually be moving.
   As Bob Marley would say: No movement, no Work.

If you have those two, then you can calculate the Work done by a particular force two equivalent ways:

The first is a formal Dot Product: $W = \vec{F} \cdot \vec{d} = |F||d|\cos\beta$, where β is the angle between d and F vectors.

The second is by calculating the component of the force parallel to the direction of displacement and then multiplying that force component by the displacement. If you use this second method, you must remember that when the force component acts in the opposite direction as the displacement, the Work is negative

Work weirdness:

(HINT: multiple choice concept questions)

Work in physics does not mean the same as work in normal everyday language usage.

For instance, if you push against an immovable wall, you do no Work in the Physics sense of the word, even though you may be pushing quite hard and may actually be getting tired. Why do you do no Work? Because the wall does not move, so the Displacement is zero, and since Work in physics is the Dot Product of Force times Displacement, anything times zero is zero. Thus, you do no Work. No movement, no work.

Here's another weird example. Say you carry an object, perhaps a bucket, while walking across a level floor. Since you are pulling straight up on the bucket (assuming it's constant velocity) and the bucket's displacement is purely horizontal, again you do no work on the bucket. Why not? From the Dot Product perspective, the angle between the Force you exert and the bucket's Displacement is 90°, and the cos(90°) is zero. Thus, the Dot Product comes to zero. If you think of Work the other way, as the amount of force parallel to the direction of displacement times displacement, there is no part of your pulling force (which is all vertical) in the horizontal direction since you're pulling straight up. So the amount of Force parallel to the direction of Displacement is zero, making Work equal to zero.

Therefore: Whenever any force acts 90° to the displacement, it does no work.

Work can be positive or negative. Positive work results when the force is at least somewhat in same direction as displacement and *negative* work results when the force acting somewhat in the *opposite* direction as displacement. Positive work thus tends to "help" motion, while negative work tends to "hinder" motion.

Answers to selected questions

1. 6 N·m
2. a. 33 N left (assuming motion right) b. 33 N right c. 56.1 N·m d. 0 N·m
3. a. 68 N·m b. 11.9 N·m
4. 685.05 N·m
5. a. 8.83 N c. 72N·m
6. 180 N·m
7. a. 68.4 N·m
8. 27.8 N·m

Work-Energy 1
Practice Questions

Draw and label a Free Body Diagram with an initial / final depiction and the displacement vector clearly identified as in the Examples above. Construct an additional data table if you find it helpful. Show all equations in letter form first.

1. A 8 Kg block is pulled by a rope across a frictionless surface to the left with a horizontal force of 3 N to the left for a distance of 2 m. How much work does the tension force do on the block?

2. You push an 11 kg case of bottled water with a horizontal force across the kitchen floor at a constant velocity for 1.7 m. The coefficient of friction between the floor and the case of bottled water is 0.3.

   a. How strong is the force of friction opposing the motion?

   b. How strong is the force with which you are pushing on the case?

   c. How much work do you do (i.e. does the pushing force do) on the case of bottled water?

   d. What is the total work done on the case of bottled water?

3. You push an 11 kg case of bottled water across the kitchen floor to the right for 1.7 m with a horizontal force 40 N to the right. The coefficient of friction between the floor and the case of bottled water is 0.3.

   a. How much work do you do (i.e. does the pushing force do) on the case of bottled water?

   b. What is the total work done on the case of bottled water?

4. Max pulls the Grinch's empty 35 kg sled across the level snow of Whoville in a straight line for 8 m. The rope from Max to the sled is at an angle of 17° above horizontal and Max pulls on the rope with a force of 135 N. The coefficient of friction between the sled and the snow is 0.14. What is the total work done on the sled?

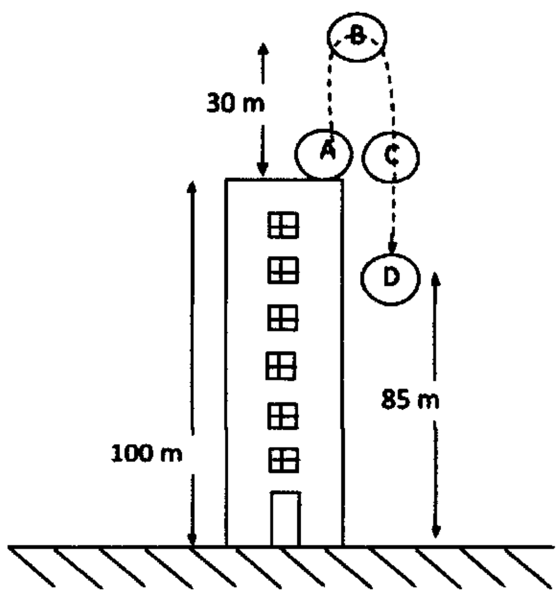
5. Sally pulls a 5 kg wagon across level ground to the left for 9 m with a rope angled 25° above horizontal at a **constant velocity**. A frictional force of 8 N opposes the motion.

    a. With how much force does Sally pull on the rope?

    b. What is the total work done on the wagon?

    c. How much work does Sally do on the wagon?

6. What is the Work done by the force of gravity on a 1.2 kg ball as the ball moves from position A to position D?



7. A 4 kg block slides 5 m down the surface of a frictionless ramp which is angled 20° above horizontal.

   a. How much work does gravity do on the block?

   b. What is the total work done on the block?